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DATA FOR GROUNDWATER MODEL STUDIES

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AND
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DATA FOR GROUNDWATER MODEL STUDIES

Part I. Model Studies Applied to Groundwater Investigations

By R.O. van Everdingen

Part II. Conditions of Similitude for Model Studies of Groundwater Flow System

By B.K. Bhattacharyya

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INTRODUCTION

The solution of many groundwater problems requires the application of analytical methods. Often a mathematical solution fails, either because actual conditions are too complicated to approximate the simplified boundary conditions implied in the theoretical equations, or because the complicated solution has to be arrived at by using an electronic computer, which may be too expensive a procedure for many of the problems involved.

In such cases it may be of advantage to try solving the problem by model studies, which are generally both cheap and readily adaptable to different problems.

Part I of this paper presents a review of model and analog studies applied to groundwater problems, compiled to facilitate the choice of the appropriate methods for solution of the various problems, and providing practical details for the application of the methods.

Part II presents the scaling factors required to simulate a groundwater flow system, with a free surface, by a laboratory model.

Part I - MODEL STUDIES APPLIED TO GROUNDWATER INVESTIGATIONS

by R.O. van Everdingen

HYDRAULIC SCALE-MODELS OR SAND MODELS

One of the oldest types of models used in groundwater studies is the sand model (e.g. Pennink, 1905). In a sand model, nature is imitated as closely as possible on a certain scale.

The main difficulty arising with sand models is caused by the capillary rise of the model liquid in the pore spaces above a free water table. The capillary zone is a part of the flow field, in nature as well as in the model. In the model, however, it forms a relatively much larger part of the flow field than in nature. As long as only groundwater is represented in the model, care can be taken that the top of the capillary zone corresponds with the upper boundary of the flow field. This is no longer possible as soon as free water (canals, rivers etc.) are present. In many cases this will result in large incongruencies.

The capillary effect can be minimized by the use of coarse sand or fine gravel and by selecting a liquid with a small capillary rise. The use of plastic grains may be of advantage.

Another difficulty is that measurements inside the flow field in the model will affect the field to a certain degree. Use of a larger scale will minimize this effect.

Sand models can be used for solution of both two- and three-dimensional problems concerning confined and unconfined aquifers.

ANALOG MODELS

Groundwater problems often require the solution of the equation of Laplace. Analytical solutions are, with few exceptions, impractical or even impossible. It is very difficult to find boundary conditions in practice that are simple enough to allow a complete particular solution to be obtained. In many cases the use of digital computers, although giving very accurate solutions, is too expensive for the problem involved.

Analog solutions, on the other hand, are generally easy to apply and inexpensive. Two apparently unrelated physical phenomena are analogous if their differential equations—each with its own boundary conditions specified in terms of independent variables—are formally identical.

Groundwater flow in homogeneous media is described by Darcy's law, electric currents in uniformly conductive media obey Ohm's law, and the flow of a viscous liquid in the interspace between two parallel plates is described by Poiseuille's law. As illustrated below, these laws are formally identical. All three of them obey the differential equation of Laplace.

Darcy's Law. $Q = K.A.\frac{H}{L}$ in which Q = quantity of water

K = permeability coefficient

A = cross-sectional area

H = potential head, producing flow

L = length of flow path

Ohm's Law. $I = \frac{1}{R}.A'.\frac{V}{L'}$ in which I = amount of current

1/R = conductivity

A' = cross-sectional area

V = voltage, producing flow

L' = length of flow path

Poiseuille's Law. $Q = \frac{b^2}{\nu} . A'' . \frac{P}{L}$ in which

Q = quantity of liquid

b = width of interspace

ν = viscosity of the liquid

A'' = cross-sectional area

P = pressure, producing flow

L = length of flow path

Thus, the phenomena of electric currents in uniformly conductive media, and flow of a viscous liquid through a narrow channel, constitute analogies that can be used to solve various problems of groundwater flow.

ELECTRIC-ANALOG MODELS

General Principles

In electric analog models, two variables are defined. The 'across' variable (a scalar value) gives the condition at a certain point of the field as compared to the condition at another point (e.g. potential difference). The 'through' variable (a vector, having both magnitude and direction) is the same in every point of an electric element and does not require a point of reference (e.g. current in a resistor). Across and through variables are related by the parameters of the system.

The basic elements occurring in electric models are: kinetic energy stores - inductors; potential energy stores - capacitors; energy dissipators - resistors. The analogy between the electro-dynamics of the model and the fluid mechanics of the groundwater prototype can be summarized as follows:

Variable or element		Electrodynamics		Fluid mechanics
Across variable	—	Voltage	—	Hydraulic head
Through variable	—	Current	—	Flow rate
Energy dissipator	—	Conductor	—	Permeability
Kinetic energy store	—	Inductor	—	Density
Potential energy store	—	Capacitor	—	Compressibility

Boundary conditions in electric models can consist of a specified voltage, a specified current, or no current source. No current implies that there is no voltage gradient normal to the boundary.

If only resistors are implied, time is not an independent variable, and so it does not appear in the characteristic equations. Final or steady-state values are reached at the instant of application of the excitation (source of energy of the model).

The same would be true if the field was entirely composed of one of the two other basic elements, capacitors and conductors. If more than one type of basic element is present in the field, steady state is reached a certain time after the last change in the excitation. Measurements must be made after steady state is attained.

Pure inductors or capacitors are difficult to obtain. They usually include a small amount of resistance. Resistors containing a negligible amount of reactance (capacitance or inductance) are easily obtainable. Therefore, electric-analog systems for the Laplace equation are almost always of the conductive non-reactive type.

If excitation occurs solely at the boundaries (external sources), all charges must return to external sources and all lines of current flow must end at the boundaries. This implies that no maxima or minima in voltage can occur within such a field, as this would generate current flow in all directions away from or toward the point of maximum or minimum voltage.

In a purely resistive field the distribution of voltage is independent of the absolute values of the resistivity.

A problem is made ready for electric analog solution along the following lines:

a. Across and through variables are identified.

b. The characteristics of the field are examined, to determine the types of basic elements present.

c. The pertinent differential equation is determined, based on the types of elements.

d. The equation is modified, to take into account any internal energy sources.

e. Boundary and initial conditions are specified.

For every second derivative with respect to a space variable, two boundary conditions must be supplied. If a first derivative with respect to time is present in addition to the Laplace term, one boundary must be added. If a second derivative with respect to time is present, two initial conditions are required. Boundary conditions are: constant potentials, or potential gradients. Initial conditions are energy stored by each energy reservoir in the field.

Many different types of electric-analog models have been used in groundwater studies so far. Electrolyte solutions or solid materials may be employed as conducting media in such models. Simplified diagrams of the usual model circuit and its bridge-of-Wheatstone principle are presented in Figure 1. Use of D.C. instead of A.C. makes no difference in the general set-up.

A distinct advantage of electric-analog models is the fact that both equipotential lines and flowlines can be represented. Thus an experiment may be arranged such that the electric equipotentials represent groundwater flowlines and vice versa (inverse analog). This obviates the need for calculating or constructing flowlines from equipotentials.

Required Properties of the Conducting Medium

It is desirable to have a conducting medium that will yield results of maximum accuracy and reproducibility. Thus the medium must possess physical properties closely approximating those assumed when specifying the analogy. Properties must be constant, both with respect to time and external conditions. Thus the medium must be: 1) homogeneous; 2) uniform; 3) isotropic as regards resistivity and current flow; and 4) unaffected by changes in temperature, humidity, atmospheric pressure, etc.

In addition, the resistivity must be of suitable magnitude to facilitate measurement. The medium should further allow easy application of boundary conditions. It should also be possible to vary the conductivity of the medium, continuously at a predetermined rate, or abruptly at a given boundary. The medium should be easily adaptable to the various configurations and forms required for the experiments.

Continuous Solid Conductors - Teledeltos Paper Models

Various continuous or semicontinuous solid media have been used, which approximately fulfil the requirements listed above.

Metal sheets or metallized paper have as their main disadvantage their low resistivity and inherent errors owing to slight variations in thickness. Woven grids of metal wire and silk thread, and ordinary drawing paper have also been used. Conductive rubber sheets, with resistivities ranging from 400 to 2,000 ohms per square, show anisotropies of approximately 10 per cent, and their resistivity is affected by mechanical stresses.

The most commonly used, continuous solid conducting medium appears to be Teledeltos paper, which is available in rolls, 36 inches wide, from various manufacturers. The resistivity of this paper is approximately 2,100 ohms per square.

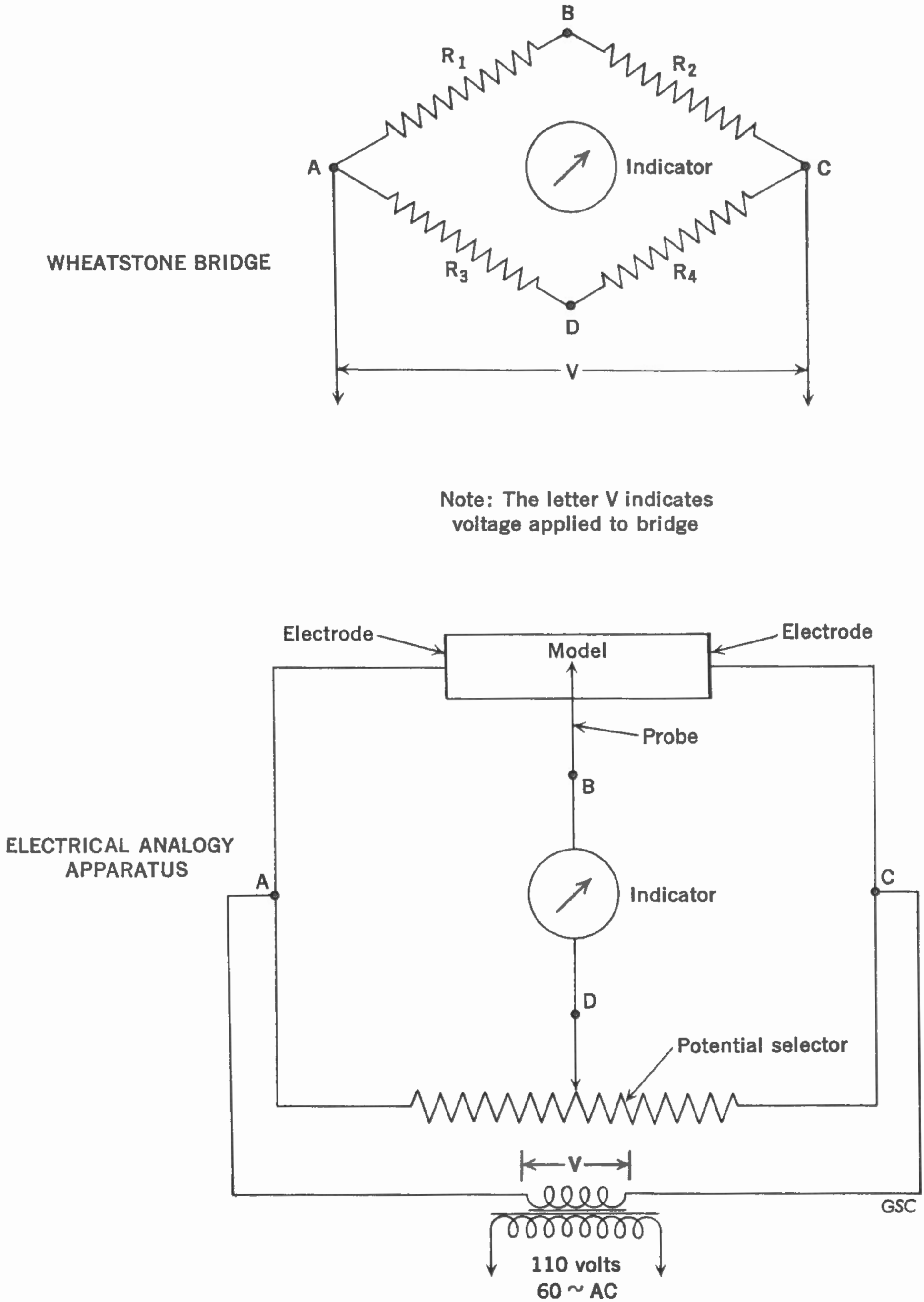

Figure 1. Simplified diagrams of an electrical analog model

Properties of Teledeltos Paper

The main properties of Teledeltos paper, which influence the accuracy of experimental results, are:

a. The variation of the conductivity in a roll of paper may be as high as 10 per cent. As only fairly small pieces of the paper are used for each experiment, however, this variation should have a negligible effect on the results.

b. The paper is hygroscopic, the humidity coefficient for the resistivity being 0.5×10^{-2} for each per cent relative humidity at 30 per cent relative humidity, and 2.0×10^{-2} for each per cent relative humidity at 80 per cent relative humidity.

c. The resistivity of the paper is temperature dependent, the temperature coefficient being $\alpha = 8.05 \times 10^{-4}/°F$, at 0 per cent relative humidity.

d. Folds and creases in the paper may change the resistivity, so care should be exercised in handling the paper.

e. When voltages exceed 40 V the paper will turn black. Furthermore, the voltage and the current density should be kept low to avoid heating the paper and changing the conductivity. Maximum energy dissipation should not exceed 0.25 watts/square inch. To keep errors due to heating below 0.5 per cent, energy dissipation should be kept below 0.013 watts/square inch.

f. The anisotropy ratio for resistivities parallel to, and across the length of the roll of paper is 1.10. The effect of this will be discussed in more detail later.

The influences of temperature and humidity make it advisable to carry out the experiments under controlled atmospheric conditions.

Test Experiments

In view of the importance of some of the potential sources of inaccuracy outlined above, it is advisable to carry out some test experiments on the paper, before starting on the actual model studies. Tests described here are derived from Rallis, Freed, and Suzman (1960).

For calibration of the paper straight equipotential boundaries are applied to two opposite edges of a square of the paper, in one case parallel to the length of the roll, in the other across it. Equipotentials at intervals of 10 per cent are plotted and drawn. Theoretical distribution of equipotentials is also constructed. The error E is determined from the perpendicular distance between theoretical and experimental equipotentials and the distance between the 0 and 100 per cent equipotentials, using the equation:

$$E = \frac{V_{theor} - V_{exp} \times 100\%}{\Delta V_{max}}$$ in which ΔV_{max} is the voltage difference between the two equipotential boundaries.

The maximum errors found in the x and y directions by Rallis, Freed, and Suzman (1960) were 0.95 and 4.08 per cent respectively. Total error in the power supply and the measuring unit was 0.56 per cent. It is advisable, in experiments related to the same problem, to use the same direction of cutting for all model sheets.

The anisotropy factor of approximately 1.10 can be determined in the same way. The factor means a difference of approximately 10 per cent between Rx and Ry. The influence of this anisotropy could be counteracted by a distortion of the model scales, so that

$$x_1 = x \text{ and } y_1 = y\sqrt{R_y/R_x} = y\sqrt{1.10}$$

In this case circular equipotential lines will become ellipses, with $x_1^2 + 1.10\, y_1^2 = r^2$. Measuring the positions on the paper in terms of x_1 and y_1 would transform the ellipses back to circles.

In the experiments described by Rallis, Freed, and Suzman (1960, p. 14), distortion of the y-scale resulted in greater inaccuracies than resulted normally. Nevertheless a scale distortion may be useful in the plotting of equipotential lines.

Adaptation of Conductivity

A change in the conductivity of a paper model can be realized in various ways. Increase of R is obtained by punching small holes in the paper. Decrease of R results when two sections of Teledeltos paper are connected electrically parallel, and also when they are glued together with graphite paint. It is a distinct disadvantage of Teledeltos paper, however, that only major changes in R are possible, and that no continuous range of resistivities is available.

Boundary Conditions

Boundary conditions are easily applied to a Teledeltos-paper model. The three kinds of boundaries that occur in paper models are:

1) Streamline boundaries are the cut-off and unconnected edges of the paper model.

2) Equipotential boundaries can best be applied by using silver paint (e.g. DuPont #4817 or Metallurgical Products Ltd. paint #21-2) or colloidal graphite (Aquadag 80). Wire connections can be attached by these paints, or by stapling them to the paper, or by using crocodile clips. Silver conducting paint is probably better, because it will reduce the voltage drop at the boundary (which may cause errors up to 3 per cent).

3) Specified varying boundary potentials are taken from a Ni-chrome wire potential divider across the low-voltage supply. Wire connections are stapled or painted to the paper. A source of error is automatically introduced here, because of the discrete points of application of the boundary potentials. Distances between the points of application should be 1/4 to 1/2 inch. It is better to use a resistance paint along the whole boundary, to

obtain linear voltage drops between the points of application. Experiments by Rallis, Freed, and Suzman (1960) showed maximum errors of 1.2 per cent between theoretical and experimental potential values obtained for a rectangular sheet.

Power Supply

For power supply alternating current is sometimes preferred, although the use of direct current may be easier, because no phase-shift effects can occur in that case.

A rheostat is often included in the circuit of the model to maintain a constant current flow in models of different resistivity.

Probes

An ordinary pencil may be used as a probe. The back end of the lead should be exposed for about 1/2 inch to make possible the connection to the sensing unit, by way of a crocodile clip. A steel ball-point pen could be useful to avoid scratching the paper.

Equipotential lines (or flowlines) may be sketched directly on the Teledeltos paper of the model, but this procedure may influence the resistivity of the paper. Therefore the use of an accurate pantograph is advisable, enabling in addition the choice of different scales of recording. If symmetry exists in the problem, plotting can be restricted to one symmetry element.

Sensing Unit

The sensing unit is generally based on the principle of zero detection. It may be a cathode-ray oscilloscope, or a sensitive 0-1 millivolt vacuum-tube voltmeter, the latter being less expensive. Earphones or a high-impedance voltmeter may be used where very high accuracy is not required.

The important point in selecting a suitable sensing device is the requirement that the internal resistivity of the instrument must be large, to prevent influencing of the potential field by electric current flow through the sensing unit. The general resistivity requirement for the whole circuit can be expressed as:

$$R_{\text{probe + sensing unit}} \rangle R_{\text{paper}} \rangle R_{\text{paint}} \rangle R_{\text{potential divider}}$$

Plotting

Equipotentials can be plotted by selecting a certain potential and moving the probe along the paper in such a way, that the sensing unit always indicates zero. When an equipotential is finished, another potential is selected and the procedure repeated. It is also possible to construct equipotentials by interpolation between potential values measured at the points of intersection by two sets of gridlines.

Flowlines must be drawn orthogonally intersecting the equipotentials; they should meet equipotential electrodes (boundaries) also at right angles. The spacing of the flowlines represents the current- or flow-density. Spacing should be such that flowlines and equipotentials form so-called curvilinear squares. A curvilinear square is an area bounded by four curved lines such that the mean 'width' of the area is equal to the mean 'length'. The tangents to the boundaries of the curvilinear square at its corners must intersect one another at 90 degrees.

Sources of Error

The conductive sheet does not simulate correctly the properties of the original field, because of inhomogeneity and anisotropy giving rise to systematic and random errors. Boundary conditions are not simulated correctly also giving rise to systematic and random errors. Inaccuracies in sensing and plotting result in random errors.

Applicability

Applications of the Teledeltos-paper models are limited to vertical and horizontal, two-dimensional problems. In some instances it may be possible to solve multi-layer problems with this method.

Continuous Liquid Conductors - Electrolyte-Tank Models

Continuous liquid conductors are electrolyte solutions. The electrolyte tank consists of a watertight container of a non-conductive material (e.g. plastic), filled with 0.5 - 1 inch of conducting liquid.

Properties of the Conducting Liquid

The conducting liquid should have the following properties:

1) no electrical resistance, or purly resistive;
2) resistivity must be uniform throughout the liquid;
3) resistivity must be of the right order of magnitude, so that the requirement $R_{\text{sensing unit}} > R_{\text{liquid}} > R_{\text{electrodes}}$ is easily fulfilled;
4) resistivity must be linear (linear relation between voltage and current amperage);
5) chemical reactions between liquid and electrodes must be impossible;
6) surface tension of the liquid must be small;
7) the surface of the liquid must stay free of films, dirt, or other foreign matter; and
8) the rate of evaporation of the liquid must be slow.

Various combinations of electrodes and electrolytes fulfil these conditions to a greater or lesser extent. Three parts per 1000 $CuSO_4$ in distilled water is an electrolyte often used. The conductivity (or resistivity) of the electrolyte should be determined if a scale factor for electric conductivity and ground transmissibility is required for the model.

Adaptation of Conductivity

Lateral variations in permeability and thickness of an aquifer can be simulated in the model by varying the depth of the electrolyte. Maximum transmissibility is represented by the full depth to the tank bottom. Lower transmissibility is represented by properly shaping the bottom of the tank so that electrolyte depth is proportional at all points to the product of permeability and thickness of the aquifer. Conductivity of the electrolyte is easily adjustable by varying the concentration of the solution.

Boundary Conditions

Potential boundaries are represented in the model by electrode strips modelled in the required forms and connected to a potential divider. Thus, they represent, on a predetermined scale, the groundwater potentials in their respective locations.

Streamline boundaries are made of insulating material.

Groundwater discharge, by pumping wells, flowing wells, or springs, simulated by current electrodes, is regulated by a milliammeter in the model.

All electrodes must have the following properties:

1) their resistivity must be negligible relative to that of the electrolyte;
2) they must not react chemically with the electrolyte;
3) they must not be dissolved to any appreciable extent by the electrolyte;
4) they must have a smooth surface; and
5) they must be easily shaped and cleaned.

Platinum appears to be the most suitable material, but excessive cost makes it more practical to use silver-plated steel, or graphited brass electrodes. The latter have proven adequate in most cases.

Power Supply

In order not to generate polarization or other disturbing chemical reactions in the electrolyte, alternating current is used exclusively. A frequency of approximately 1,500 cycles per second seems to be the most satisfactory for avoiding both surface- and stray-capacitance effects. A stabilized low-voltage power supply of the required frequency can be provided by a medium-to-high frequency generator.

Probes

Groundwater potentials, as simulated by electric potentials, are measured in the electrolyte with a probe connected to a sensing unit. The tip of the probe should be as short and thin as possible, to avoid distortion of the potential field through the measurement. Formation of a capillary meniscus, distorting the field around the probe, could be prevented either by adding a wetting agent to the electrolyte, or by selecting a suitable insulating material for the probe. Softky and Jungerman (1952) used probes made of 30 mil diameter tungsten rod, insulated by several layers of Formvar (total thickness amounting to 1 mil), with a spherical tip of approximately 50 mil diameter. They mention an error of 0.2 per cent for three-dimensional potential measurements.

Sensing Unit

The sensing unit must fulfil the same requirements as for the Teledeltos-paper models.

Plotting

Plotting of equipotential lines with the electrolyte-tank model presents no special problems. A special arrangement for convenient plotting has been described by Moody and Phillips (1955). They used a scanning device, the arrangement of which is as follows. Two fixed parallel guiding bars, A, which are adjustable in height for levelling purposes, carry a movable cross-assembly B. In turn, B carries a mount, C, which holds the probe needle. C can move along B. The scales E and F, fixed to B and A, respectively, provide a coordinate system for measuring movement in the two perpendicular directions. The entire scanning device is mounted on a rigid table.

The procedure to survey an equipotential line with this device is as follows: The potential selector unit (see Figure 1) is set for the required potential. The scanning device is then set at some coordinate, either on the E or on the F scale. The other coordinate of the point on the equipotential line is determined by moving the probe (with mount C) along the other coordinate axis, until the circuit is in balance (indicator at zero). The coordinates are then plotted on a scale-drawing of the model. The scanning device is moved to a new coordinate and the procedure repeated. Spacing of coordinates is so chosen, that a smooth curve may be drawn through the plotted points. For straight lines spacing can be wider than for strongly curved equipotentials. When an equipotential is surveyed along its entire length, the selector is set to another potential, and a new equipotential line is surveyed, in a similar way.

If the potential at a certain point is required, the probe is placed on the point in question and the selector is adjusted until the indicator shows that the circuit is in balance. The potential value at the point can then be read from the setting of the selector switches.

Sources of Error

The accuracy of electrolyte-tank models is generally much greater than that obtained with conductive sheet analogs. It is not influenced by lack of uniformity, homogeneity, or anisotropy. All sources of error occurring are of similar order of importance. They include the following.

1) Mechanical errors in the shaping and positioning of tank walls and electrodes; in positioning the probe; in the depth of immersion of the probe; and in recording the position of the probe. In general, an accuracy to within e% in the results can be attained by keeping the error in the preparation of tank and electrodes within e% of the mean size of the model.

2) Polarization and phase shift effects. These can be minimized by using etched and graphited brass, or platinized platinum electrodes. Probe leads should be well shielded. Phases at the probe and at the measuring unit should be balanced.

3) Effects of surface tension can be minimized by using a short, thin probe, and by bringing the liquid level in the tank to coincide with the flat top of the electrodes. (The same effect can probably be obtained by adding a wetting agent to the liquid, or by treating the top part of the electrodes with a water repellant.)

4) Non-uniformity of the liquid can be prevented by ensuring thermal equilibrium in the system, by preventing sudden changes in temperature, and by thorough mixing.

Careful minimizing of the above sources of errors can make possible accuracies to within 0.1 per cent in two-dimensional liquid analogs, as shown by Einstein (1951).

Applications

Applicability is limited only by the fact, that resistivity is constant throughout the model. As long as isotropic conditions prevail in the prototype, horizontal, vertical, and three-dimensional problems can be solved with electrolyte models.

Some special applications, as described by de Jong (1962b), are presented here as an illustration.

1) In experiments to determine the influence of artificial measures on the groundwater regime, only the secondary flow field is simulated, i.e. only the changes in groundwater level are represented in the model. An example of this is a new drainage canal with a fixed water level lower than the present local groundwater level. The future decrease in potential at the site of the new canal is represented by electrodes which have a potential equal to the difference between present and future water levels. Rivers and other open water in which no decrease of (average) level is expected, are represented by zero equipotential electrodes. In the electrolyte the potentials are measured representing the future lowering of the water levels in the model area. The future total water levels can then be calculated from present levels minus the measured decrease.

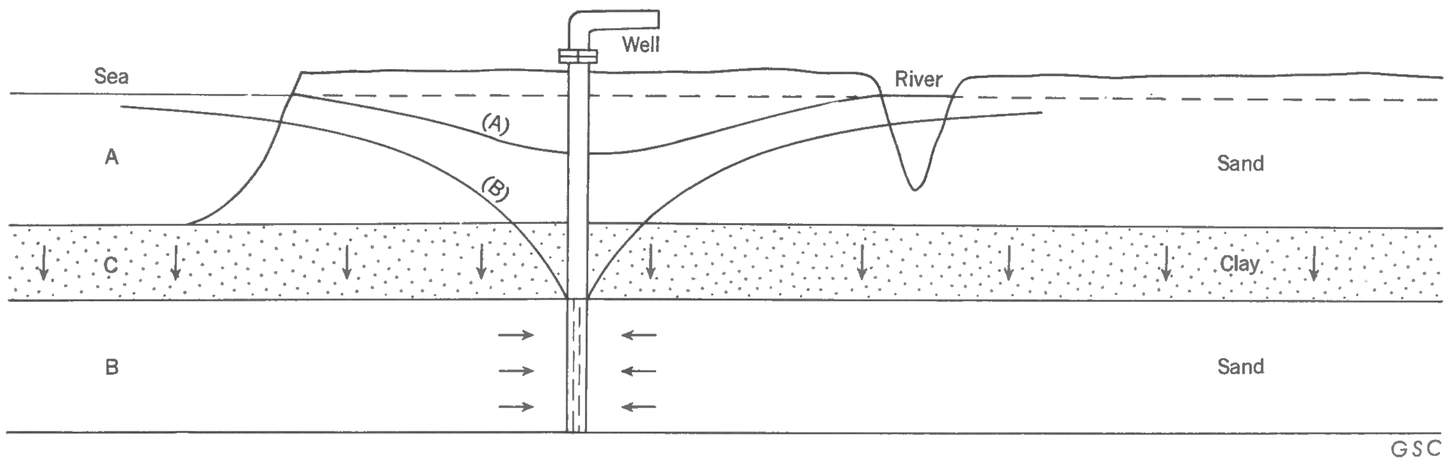

Figure 2. Prototype of a three-layer problem. (after de Jong, 1962b)

2) Besides purely confined or purely unconfined conditions, it is also possible to simulate situations in which two permeable layers are separated by a semi-permeable (or leaky) layer (Figure 2). Solution of this three-dimensional problem was first tried by using a perforated paraffin layer as the leaky aquiclude in the model. Clogging of the holes, however, soon changed the "resistance" of this aquiclude. In 1960 de Jong found the following solution to this problem: two separate electrolyte tanks (the two aquifers) were coupled by way of a large number of electrically parallel resistors (the leaky aquiclude). See Figure 3. Different permeabilities in the two aquifers can be simulated by using different concentrations of electrolyte or different depths of the same solution.

Gelatin-Electrolyte Models

An interesting variant of the electrolyte-tank models is described by Muskat (1949, pp. 670-682) for the simulation of water injection in a producing oil field. Similar problems are the (artificial) recharge of a producing aquifer, and the intrusion of seawater in an aquifer.

The aquifer here is represented by a piece of blotting paper saturated with an electrolyte solution. The solution should contain phenolphtalein, which turns red in the presence of OH ions. When the injection or intrusion electrodes are made negative, the incoming water is represented by OH ions, and its progress in the aquifer will be simulated by the advance of the boundary between coloured and uncoloured electrolyte in the blotting paper. Photographing the paper at fixed time intervals will produce a permanent graphic history of the recharge or intrusion process.

A later modification of this technique uses an agar gelatin film, such as 0.1 N zinc-ammonium chloride solution containing 1 per cent agar, deposited in a 1/16 inch layer in a transparent mould forming the areal boundaries. Well electrodes are plastic tubes (1/2 inch diameter, 1 1/2 inches long), set in a plastic cover plate and terminating in tips that penetrate into the gelatin film. The positive injection electrodes are filled with 0.1 N copper-ammonium chloride containing 1.5 per cent agar, the negative production electrodes contain the same solution as the gelatin film, but with 1.5 per cent agar. The injection electrodes are connected to the positive side of a D.C. voltage supply (up to 1.000 volt), through a bank of milliammeters with rheostat controls. The production electrodes are similarly connected to the negative side of the power supply. The blue coloration of the gelatin, developed in this case by the advance of copper-ammonium ions, is photographed through the transparent underside of the model.

Opsal (1955) described a gelatin-electrolyte model in which the electrolyte consisted of 10 parts water, 1.5 parts gelatin, and 1.5 parts glycerine. Salt was added in various amounts to regulate the conductivity of the mixture. Some beta naphtol was used to prevent decay of the solution. A mixing temperature of 100° to 125°F was recommended. Such models seem to be specially suited to the study of multilayer and three-dimensional problems. Lateral variations in thickness and permeability in one aquifer are again simulated by making the depth of the gelatin film in any point of the

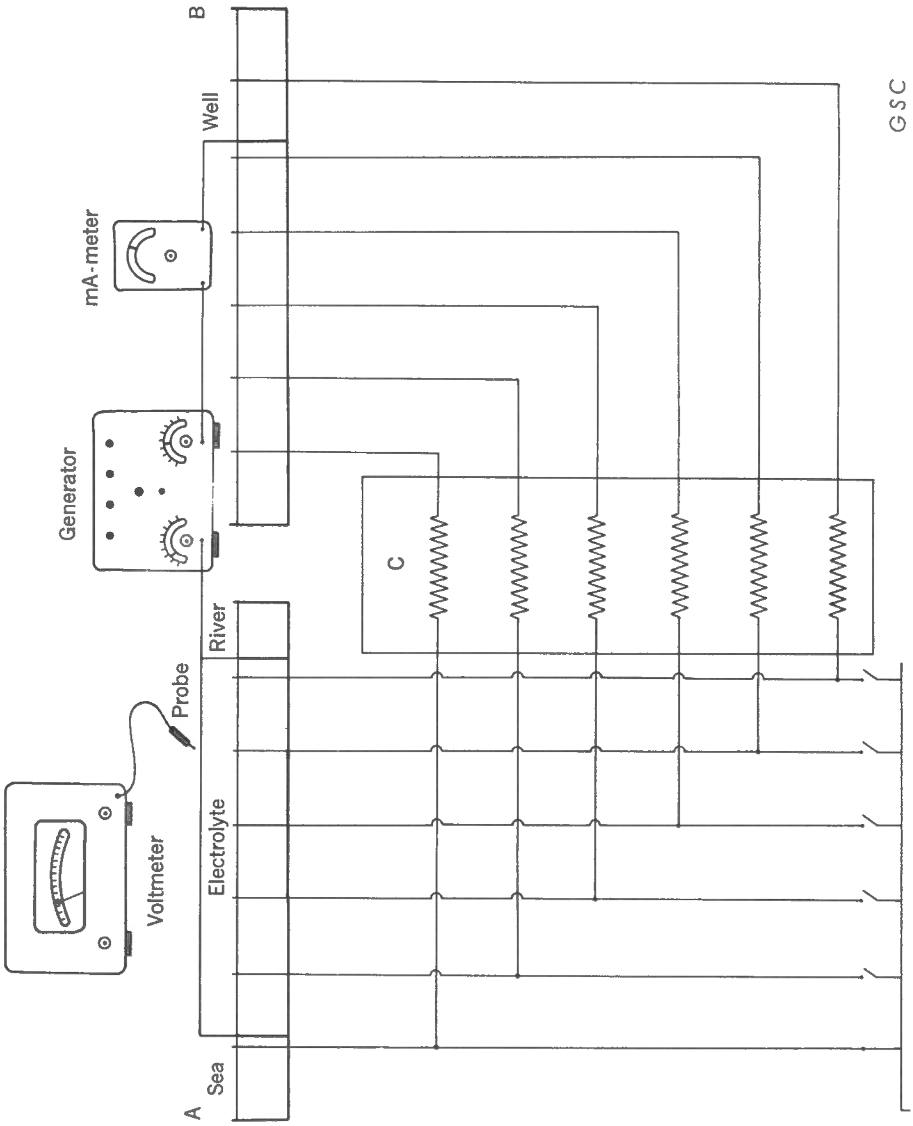

Figure 3. Wiring diagram of the model for a three-layer problem. (After de Jong, 1962b)

model proportional to the product of hydraulic permeability and thickness of the aquifer, in the corresponding part of the prototype. Aquifers of different permeability can be represented by gelatin layers of different electric conductivity.

Use of A.C. power is essential to avoid polarization effects.

Again, only steady-state problems should be investigated with this type of model.

Simulation of an Infinite Potential Field

Geometric boundaries of a model very often do cause difficulties, because the current field in the model is abruptly interrupted here. This interruption of the current field is allowable only when the current at the boundary does not have a component normal to the boundary. In practice, at least one boundary will not fulfil this condition, unless the model is extended beyond the immediate area under investigation. This, however, entails practical difficulties, because the scale of the model must be large enough to permit accurate measurements, while on the other hand the model must be of reasonable size. If only potential currents occur in the field under investigation, the problem of model boundaries can be solved elegantly as indicated by de Jong (1962a).

A potential current in a field with a certain coordinate system will keep its characteristics when that system is submitted to a "conformal transformation". All angles in the original field preserve their magnitude after the transformation. The principle of the transformation is the following (see Figure 4):

A point, P, outside a circle of radius R, at a distance r from the centre of the circle can, through a conformal transformation, be represented by a point, P', inside the circle, at a distance r' from the centre, such that

$$r \times r' = R^2 \qquad \text{(see Figure 4a).}$$

An infinite field with rectangular coordinates, containing a circle (Figure 4b), is represented through this conformal transformation, by two circles of radius R (Figure 4c). The circle on the left of Figure 4c represents the circular part of the original field, the one on the right a distorted picture of all the remainder of the original field. The centre of the latter circle represents the circle with infinite radius of the original field. All originally rectangular coordinates are transformed into systems of orthogonally intersecting circles.

Figures 4a and 4c present, respectively, the original and transformed configurations for a rectangular model area in an infinite field.

Through this transformation it is possible to simulate any required potential field in a simple model consisting of two interconnected parts. One part represents the area under investigation on a normal scale, the other part simulates the whole of the surroundings. By making an adequate number of connections between the two model parts, a very large accuracy of results is possible. The method has been used both with Teledeltos paper and with the electrolyte tank.

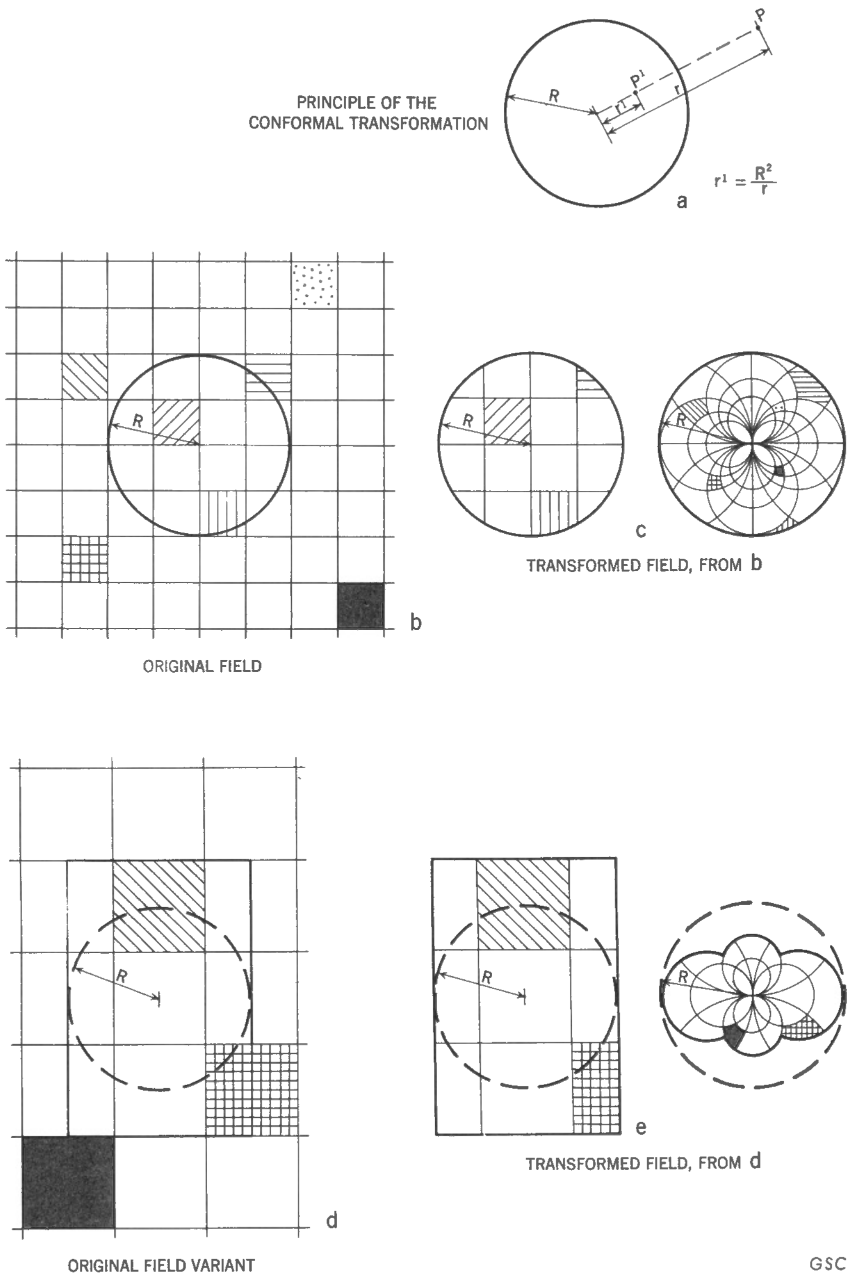

Figure 4. Conformal transformation; (from de Jong, 1962a)

Non-Continuous Electric Analogs - Electrical Network Models

Electrical network models are based on the principle of discretization of smaller or larger parts of a field, in the form of lumped electrical elements. Pure resistance (R) networks represent fields, which are described by Laplace's differential equation

$$\nabla^2 \phi = 0$$

Resistance-capacitance networks (RC) are analogs of fields described by:

$$\nabla^2 \phi = k \frac{\partial \phi}{\partial t}$$

Resistance-inductance-capacitance (RLC) networks represent fields described by:

$$\nabla^2 \phi = k_1 \frac{\partial^2 \phi}{\partial t^2} + k_2 \frac{\partial \phi}{\partial t} + k_3 \phi$$

Applications of these networks are almost unlimited. Two- and three-dimensional, steady and non-steady state problems can be handled by electrical network analogs. The accuracy of results obtained with such models is influenced by the spacing of the elements in the network. For this reason spacing is often made smaller in critical areas of the model.

Variations in aquifer characteristics, both vertical and lateral, can be simulated easily in three-dimensional networks.

Karplus (1958, pp. 170-229) presents an exhaustive review of the possibilities in this field of analog simulation. As these types of analogs are too advanced for ready application in the early stages of model studies, they are not treated further here. Electronic analogs (Karplus, pp. 229-265) are also outside the scope of this review.

Scale Factors for Electric-Analog Models (Continuous Conductor Type)

In the construction of electric-analog models, the space dimensions are generally scaled first. The scale factor employed depends in many cases on the dimensions of the available equipment. Every point in the electric analog must correspond to a specific point in the original field. Generally, geometric similarity is retained in the model, although conformal transformations (Karplus, 1958, pp. 53-77; de Jong, 1962b) may be used to great advantage in two-dimensional models. In three-dimensional models it is often convenient to select different scales for the horizontal and the vertical directions.

The model must be large enough to enable accurate measurement. Plotting may be facilitated by the use of a pantograph.

Scaling of the field parameters is only necessary when the field under investigation is non-uniform, or anisotropic. In such cases the relative magnitude of the electrical resistivity in the various portions of the analog must correspond to the relative magnitude of the field parameters in the corresponding areas of the prototype.

First, a convenient base value R_o is selected, keeping in mind the required range of field resistances, magnitude of boundary resistivities, and characteristics of the probe and the sensing unit. The choice is limited by the available conducting sheets or electrolytes and the depth of electrolyte tanks.

In three-dimensional models, variations in field parameters may be combined with variations in geometric dimensions.

No further scaling is required when all boundaries are either equipotentials or streamlines. Ratios of field potentials to boundary potentials are equal for corresponding points in model and prototype.

The scaling problem is somewhat more complex when currents are applied to the model, e.g. when a potential gradient normal to a boundary is specified, or when there is internal excitation. Resistance, voltage and current, simulating transmissibility, flow potential and flow rate, respectively, are related according to Ohm's law, which is analogous to Darcy's law. Selection of scale factors for any two of these field parameters automatically determines the scale factor for the third one.

If scale factors N_ϕ and N_T are selected such that

$$\text{Voltage } V = N_\phi \times \phi \text{ (potential)}$$

$$\text{Resistance } R = N_T \times T \text{ (transmissibility)}$$

then:

$$\text{Current } i = N_q \times q \text{ (flow rate)}$$

in which:

$$N_q = \frac{N\phi}{N_T}$$

As the distribution of current in the model usually cannot be predicted, it may be necessary to alter the scale factors initially selected, based on the results of test measurements on the model.

Advantages and Limitations of Electric-Analog Models

1. Fields can be completely confined to closed circuits and conductive media.

2. Electric fields are independent of gravity and other external fields, provided sufficient shielding is available. The levelling and aligning problems encountered with viscous flow models are thus generally absent.

3. Continuous electric conducting media can be made in the form of liquids, so that probes can be inserted into their interior, for the measurement of three-dimensional fields, and for the application of internal sources of excitation.

4. Electrical and electronic instrumentation of high quality is readily available, well standardized and easy to use. The power supply-, control-, and measuring-equipment for electrical models is fairly

expensive. If selected with care, however, it can be used for most of the different types of electric-analog models. The electric model itself, which often is used only once or twice, is in most cases inexpensive.

5. Relatively little technical skill is required to build and operate electrical models, with the exception of models simulating non-steady conditions. Viscous-flow and sand models, on the other hand, require precision workshop construction to ensure adequate accuracy of the results.

6. Virtually all groundwater field problems are capable of successful treatment by electric analogs.

7. The main disadvantage of the electric-analog method is, that a phreatic surface must be located in the model by the trial-and-error method.

Of the continuous electric analogs the conductive liquid (or electrolyte) systems are the most important. They are capable of very high accuracies, although elimination of errors of less than 1 per cent requires very careful design. They may be used for fields with equipotential boundaries, fields with a constant potential gradient normal to a boundary, and fields with internal excitation. Three-dimensional problems require special sensing mechanisms. Field parameters can be varied by variation of the depth and conductivity of the electrolyte layer(s).

A disadvantage of the electrolyte method is that for each new problem a new electrode configuration may have to be modelled. In some cases of this kind the use of Teledeltos paper may be of advantage.

Conductive-sheet analogs (Teledeltos paper) are often easier to construct than an electrolyte model, but their accuracy is limited by non-uniformity and anisotropy of the available conductive sheets (not better than 2 per cent). Plotting may be done directly on the conductive sheet. They are suitable only for two-dimensional fields. The solution of two- or multi-layer problems is virtually impossible with Teledeltos paper, because inter-connection of the layers is very difficult. Furthermore, the accurate regulation of the resistivities of the various layers is impossible with the restricted choice of resistivities available in Teledeltos. Local variations in field parameters can be simulated by perforating the sheet for an increase of R (giving, however, distortion of the field), or by pasting sheets together for a decrease in R, or by applying different resistivity paints.

VISCOUS-FLOW-ANALOG MODELS

Scale models based on the analogy between the laws of Darcy and Poiseuille, in their simplest form consist of two closely spaced parallel transparent plates, forming a narrow channel through which a viscous liquid can flow. Earliest models of this type were built in 1948. Santing (1951) presented a description of the method, with some practical examples.

The Viscous Liquid

The ideal model liquid for viscous-flow models should have the following characteristics:

1) viscosity must be easily adaptable by thinning of the liquid;
2) the temperature coefficient of the viscosity must be as small as possible;
3) the liquid must be easily dyed with a non-adhesive colouring agent;
4) Solution of salt or another chemical must enable increase in the density of the liquid;
5) the liquid must be non-hygroscopic and as independent of the relative humidity as possible;
6) the liquid must not age under the influence of light or air; and
7) the model must be easily emptied and cleaned after each experiment.

Vertical Sheet Models

When put in a vertical position and closed at the bottom, the viscous-flow or sheet model may represent a cross section through a phreatic aquifer with two-dimensional groundwater flow. In this case, the interspace between the plates represents the permeable aquifer, the liquid in the interspace the groundwater.

The two-dimensional flow field in the interspace liquid simulates the potential flow field in the two-dimensional section of the prototype aquifer. Open water (rivers, lakes, etc.) are represented in the model by a widening of the interspace.

Rainfall is imitated by sprinkling of liquid over part or all of the length of the interspace. Variations in water level can be effected by means of adjustable reservoirs connected to the interspace. Variations of hydraulic conductivity (or permeability) of the ground are represented by varying width of the interspace.

Layers of very low hydraulic conductivity (leaky aquicludes) may also be represented in the model by a strip of metal that fills the width of the channel over the required length of the layer. The strip is provided with vertical grooves simulating the vertical permeability of the aquiclude.

The closed bottom of the model often represents an impermeable boundary. In order to fulfil other boundary conditions, it is necessary to add or withdraw liquid at the closed sides (and sometimes at the bottom) of the model.

"Storage capacity" of the ground above the water table is represented by the storage capacity of the interspace above the level of the liquid. Fluids of slightly different density can be used to imitate fresh and mineralized groundwater.

Horizontal Sheet Models

In a horizontal position the viscous-flow model may simulate a certain part of an aquifer, either phreatic or confined. The interspace represents the aquifer, the value of the transmissibility being simulated by the width of the interspace. Lateral variations in transmissibility may be introduced in the model by varying the width of the interspace. Storage capacity can be added by means of vertical tubes or vessels on top of the upper plate connected to the interspace. Lateral variations in storage capacity may be effected by variations in the spacing or the diameter of the vessels.

Recharge and discharge can again be simulated by adding or withdrawing liquid at the appropriate points of the interspace.

In the case of a phreatic aquifer the capacity of the storage vessels must correspond (on a scale that will be defined later) with the "storage capacity" of the ground above the water table (the effective porosity). In the case of a confined aquifer it should correspond with the elastic storage capacity of the aquifer. The level of the liquid in the vessels in both cases represents the groundwater potential. Thus the vessels can be used as observation wells for water level measurements.

The tubes of the distribution system, for adding or withdrawing of liquid, are located under the model, connected to the interspace through the lower plate at a large number of points. They are placed under the model to keep the area above the model free from obstructions, to facilitate measurement. The connections of the distribution system are best placed opposite the storage vessels, to avoid horizontal flow causing undesirable deviations from the true potential field.

The amounts of liquid supplied or withdrawn can be regulated by pressure regulators or by valves with a manometer. Capillary tubes are designed in the lines of the distribution system, to keep the pressure head, required for flow through that system, high in comparison to the variations in pressure head in the interspace liquid.

Scale Factors for Viscous-Flow Analog Models

The relations between the model scale factors follow from the requirement that the ratio between any term in the general differential equation for the groundwater flow and its equivalent in the equation for the viscous flow in the model must be a constant.

The general differential equation for two-dimensional non-steady flow in a phreatic aquifer having recharge, is as follows:

$$T\left(\frac{\partial^2\phi}{\partial x^2} \quad \frac{\partial^2\phi}{\partial y^2}\right) + R = S\frac{\partial\phi}{\partial t} \quad \ldots\ldots\ldots\ldots (1)$$

in which: T = transmissibility

ϕ = groundwater potential

x, y = horizontal coordinates

R = recharge

S = storage capacity

t = time

For two-dimensional non-steady flow in a confined aquifer the same relation applies, when R is made equal to zero.

The corresponding equation for the model has the same appearance; all entities having the index "m", referring them to the model.

The transmissibility of the model may be written as:

$$T_m = \frac{\rho g b^3}{12 \nu} \qquad \text{..........(2)}$$

in which: ν = dynamic viscosity of the model liquid

ρ = density of the model liquid

g = acceleration of gravity

b = width of the interspace

The storage capacity of the model, S_m, may be defined as the amount of liquid released from the aquifer by a unit decline of potential head. Thus, S_m is the ratio between the area of the storage vessels and the area of the model plates. When r is the inner diameter of a storage vessel, and when each vessel represents the storage capacity for an Area A:

$$S_m = \frac{\pi r^2}{A} \qquad \text{..........(3)}$$

The relationship between the scale factors of the model now follow from equation (1):

$$N_T \frac{N_\phi}{N_x 2} = N_T \frac{N_\phi}{N_y 2} = N_R = N_S \frac{N_\phi}{N_t} \qquad \text{..........(4)}$$

where N indicates the scale, or model/prototype ratio (e.g. N_t is the time scale). We have here three relations between seven scale factors, so four of them may be chosen freely. Those four will generally be N_t, N_ϕ, N_x, and N_S.

The time scale N_t is chosen in such a way that sufficient time will be available for measurements and other operations during experiments. It will depend to a certain degree on the duration of the phenomenon to be represented. Usual values of N_t are between 1 year = 2.5 minutes and 1 year = 25 minutes.

The potential scale N_ϕ is determined by the required accuracy of the level readings. It is commonly in the order of 1:100. For vertical models the vertical scale should be the same as the potential scale N_ϕ, especially when phreatic water is represented in the model.

The geometric length scale N_x (and N_y for horizontal models) is conditioned by the requirements that the model should have manageable dimensions, while adequate imitation of actual conditions should also be possible. Usual scales N_x range from 1:300 to 1:10,000; the size of a model should not be larger than 10 feet.

The storage scale N_S must permit diameters and spacing of storage vessels, which are justified from both the hydraulic and the constructional point of view. Large diameters increase difficulty in reading; a too small number of storage vessels would result in large deviations from the correct flow field. Diameters of 0.6 inch, and spacings of approximately 4 inches gave satisfactory results.

After these scales have been fixed, the others are computed from equation (4).

In vertical models in which a definite difference between horizontal and vertical permeability for various layers must be represented, the scales for k_x and k_z can be derived from:

$$\frac{N_{k_x}}{N_x 2} = \frac{N_{k_z}}{N_z 2} \qquad \ldots\ldots\ldots\ldots(5)$$

This implies that, in order to represent actual ratios of k_x and k_z, the scales of N_x and N_z (N_ϕ) are no longer independent. The width of the interspace for the most permeable layer must then be chosen as a free scale factor.

Besides the abovementioned, scale factors are also needed for discharge and volume. The discharge scale N_Q can be derived from any discharge formula, e.g.:

$Q = B.T \frac{d\phi}{dx}$ indicating the discharge across a section of the aquifer of length B.

$$N_Q = N_y . N_T \frac{N_\phi}{N_x} = N_T . N_\phi \qquad \ldots\ldots\ldots\ldots(6)$$

The volume scale N_V is given by:

$$N_V = N_Q . N_t \qquad \ldots\ldots\ldots\ldots(7)$$

For phreatic conditions, the recharge scale is:

$$N_R = \frac{N_Q}{N_x . N_y} \qquad \ldots\ldots\ldots\ldots(8)$$

The width of the interspace, b, and the model liquid to be used, have to fulfil the following special conditions:

1) the values of b, ρ, and ν (at the average temperature during the experiments) must satisfy equation (2); and

2) flow through the interspace must be laminar, implying that the Reynolds number has to be smaller than about 1.000.

Several liquids and a range of values for the width of the interspace fulfil these conditions. When a number of additional practical requirements is taken into account, the most appropriate values for ρ, ν, and b are easily determined. Two of these practical requirements are:

1) b cannot be made too small, as this will entail difficulties and errors through imperfections in the two parallel plates that form the interspace. Usual values for b are between 0.8 and 2 mm; and

2) the viscosity of the model liquid must be affected as little as possible by changes in temperature, humidity, etc.

In any case, temperature must be kept as constant as possible during the experiments; a change in the viscosity of the model liquid automatically changes the time scale N_t, as well as the transmissibility scale N_T.

When problems involving sweet and mineralized water are to be studied, the scale for the difference in density, N_Δ, has to be equal to unity. Otherwise the scale for height N_z and the scale for potential N_ϕ in a vertical model would become different. Viscosities of sweet and salt water are virtually the same in nature, and they should also be the same in the model.

To verify the often inadequately known hydrologic parameters, the actual conditions during a number of recent years are imitated and the results compared with data obtained in the field. Adjustments in the model scales can then be made on the basis of this comparison. After this, planned or anticipated development can be simulated with the model to obtain information about future conditions.

Accuracy and Limitations of Viscous-Flow Analogs

1) The accuracy of results from viscous-flow analogs depends on the technical precision of the model, on the scales, and on the amount of simplification applied to subsurface conditions. For simple problems, such as flow of groundwater in a homogeneous layer, errors in the results for water levels and water quantities should not exceed a few per cent. In more complicated problems, errors as high as 20 per cent have been found in the results.

2) Storage capacity for the horizontal model is not distributed uniformly over the entire area of the aquifer; it is concentrated in a restricted number of storage vessels. Therefore, a non-steady flow through the aquifer, when simulated in the model, will produce additional flows through the interspace, from or to the storage vessels. These flows are superimposed on the original flow field. Closer spacing of the vessels will reduce such deviations.

3) Deviations from correct potentials are largest in the immediate vicinity of points of recharge or discharge. The potential flow field in these points cannot be reproduced correctly in the model. Therefore, boundary conditions related to these places should preferably be expressed in quantities of liquid, rather than in groundwater potentials.

4) After the model is assembled, the interspace is fixed, which means that the value of transmissibility, although it may vary from place to place, is also fixed. Thus, in simulating water-table conditions in a vertical model, the variations in water level must be negligibly small as compared to the total thickness of the aquifer, in order not to affect the value of transmissibility to any large extent.

5) Applications of viscous-flow models are restricted to two-dimensional problems.

SUMMARY AND CONCLUSIONS

Both sand models and viscous-flow models require precision construction, which may make them expensive. Besides, each new problem generally necessitates alterations in the model, to be carried out with the same precision as the original construction. Where transitory effects are of major interest, the use of these models is justified.

Sand models can be used for three-dimensional problems involving non-steady flow.

Continuous-conductor electric models are constructed from ready-made electrical components. The power-supply and measuring instruments are fairly expensive, but they can be used for different types of electric models. The actual model, either made or adapted for each new problem, is composed of inexpensive Teledeltos paper or an electrolyte solution, a number of connection wires, and in some cases, a number of electrical resistances. When electrolyte-tank models are used, a new electrode configuration must be made for each different problem.

From the point of cost, Teledeltos-paper models are preferable. Electrolyte models may be slightly more expensive. However, the range of applications of, and the accuracy of results obtained with electrolyte models, are superior to those of Teledeltos paper. Which method is selected depends on the type of problem, on the desired accuracy, and on the available equipment.

The more complicated electric-network models have a large range of applications and they may give very accurate results. Their use in the early stages of model studies, however, is not recommended, because of their greater complexity.

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Part II - CONDITIONS OF SIMILITUDE FOR MODEL STUDIES OF GROUNDWATER FLOW SYSTEM

by B.K. Bhattacharyya

In this report attempts will be made to determine the conditions of similitude for representing the groundwater flow system with a free surface by a laboratory model.

The concept of pressure in an incompressible fluid, such as groundwater where the density is not dependent on pressure, presents certain difficulties to the physical understanding. The pressure is considered a variable of state, but is denied any influence upon the way in which the fluid occupies the space. The fluid has a free surface when it is bounded by vacuum or air. The condition for the free surface (Sommerfeld, 1950, p. 169) is

$$p = 0$$

i.e., the atmospheric pressure is taken as zero.

If the hydraulic head 'h' is taken as the height of any point with respect to a reference level, the velocity at that point along a streamline is given by (Scheidegger, 1960; Muskat, 1949)

$$\vec{v} = -\frac{K}{P}\ \text{grad}\ h \qquad \text{..........(1)}$$

where K is the hydraulic conductivity, and P the porosity of the soil, a dimensionless quantity. The hydraulic conductivity K is equal to $k\rho g/\mu$ where k is a property of the soil called permeability and expressed as sq. cm., ρ the density of the fluid, g the acceleration due to gravity, and μ the viscosity of the fluid. K has the dimension of length upon time.

The free surface is a streamline under steady-state conditions. Hence the velocity of the free surface is given by

$$v_f = -\frac{K}{P}\ \text{grad}_n h \qquad \text{..........(2)}$$

where $\text{grad}_n h$ denotes the normal component of the gradient of the hydraulic head at the free surface.

Let the length and time parameters be denoted by

$$\text{Length} = l_o L$$

and

$$\text{Time} = t_o T$$

where the unit quantities of length and time are l_o and t_o, whereas the dimensionless measure numbers of the parameters are L and T.

If we assume K_o/P_o as the unit quantity of the parameter K/P we have

$$\frac{K}{P} = \frac{K_o}{P_o}\left(\frac{\overline{K}}{\overline{P}}\right) \qquad \text{..........(3)}$$

where $(\overline{\frac{K}{P}})$ is the dimensionless measure number of the variable (K/P).

Equation (2) may, therefore, be written as

$$\overline{v}_f = - \frac{t_o}{l_o} \cdot \frac{K_o}{P_o} \, (\overline{\frac{K}{P}}) \text{ grad }_n h \qquad \text{..........(4)}$$

where $\overline{v}_f$ is the measure number of the velocity. The expression grad $_n$h will remain unchanged because this is a dimensionless quantity.

In order that the flow pattern of the groundwater in the field remains similar to that in the model, it is necessary and sufficient, subject to the restrictions imposed on the validity of (1), that the coefficient (t_oK_o/l_oP_o) be identical in both the systems. In other words, the characteristic parameter C in

$$\frac{t_o}{l_o} \cdot \frac{K_o}{P_o} = C \qquad \text{..........(5)}$$

should be invariant to a change of scale.

If we reduce the length parameter by a factor of 1000, say, we have

$$\left(\frac{t_o K_o}{P_o}\right)_f = 1000 \left(\frac{t_o K_o}{P_o}\right)_m$$

where the subscripts f and m denote the values of the parameters in the field and in the model respectively. Furthermore, if the soil and fluid properties remain unchanged,

$$(t_o)_f = 1000 \, (t_o)_m,$$

i.e., any phenomenon occurring in the model will take place one thousand times faster than in the field. Thus, the time scale is reduced by the scaling factor of length provided that the soil and fluid properties remain fixed.

Now if $l_f / l_m = r_l$, and

$$\frac{(K_o/P_o)_m}{(K_o/P_o)_f} = r_k, \text{ we have}$$

$$(t_o)_f = r_l \, r_k \, (t_o)_m. \qquad \text{..........(6)}$$

Utilizing equation (6), we may be able to achieve a considerable reduction in the ratio of the time scales in the model and in the field.

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